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Numerical Solution of Acoustic-Structure Interaction Problems

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This report describes a technique for solving steady coupled structure-acoustics problems. This technique is applicable to both structural radiation problems and structural scattering problems.

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I INTRODUCTION

The purpose of this report is to describe a technique for solving steady state coupled structure-acoustics problems that has been implemented by the transducer modeling group (Code 712) at the Naval Ocean Systems Center (NOSC). This technique is applicable to both structural radiation problems and structural scattering problems.

The structural portion of the problem is modeled using a standard finite element program. The particular program used at NOSC is MARTSAM, but virtually any general finite element code could be used. The acoustical portion of the problem is modeled using the Combined Helmholtz Integral Equation Formulation (CHIEF)[1]. This technique has been implemented at NOSC in a computer program that is also called CHIEF[2]. The original documentation on CHIEF described only its acoustic radiation capabilities, but the capability to do scattering problems has been implemented and is described in the revised CHIEF Users Manual. The coupling of the structural and acoustic models is performed in a separate program that combines certain outputs of the finite element and CHIEF programs. The approach that has been implemented follows that described in Wilton[3] except for the way in which the continuity of normal velocity is enforced.

This report is organized as follows: Section II summarizes the basic finite element structural relations; Section III summarizes the basic CHIEF acoustic relations; Section IV describes the combination of the CHIEF and finite element results to solve the coupled problem; Section V describes the output quantities that are computed for a scattering problem, and compares numerical results for scattering from an elastic spherical shell to the analytical solution; and Section VI describes the output quantities that are computed in a radiation problem, and compares numerical results for radiation from a locally driven elastic spherical shell to the analytic series solution. Section VII contains a summary of the features of the model described in this report.

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[1] H. Schenck, *Improved Integral Formulation for Acoustic Radiation Problems*, J. Acoust. Soc. Am. 44, 41–58 (1968).

[2] G.W. Benthien, D. Barach, and D. Gillette, **CHIEF Users Manual**, Naval Ocean Systems Center Technical Document 970, Rev. 1, November 1988.

[3] D.T. Wilton, *Acoustic Radiation and Scattering from Elastic Structures*, International Journal for Numerical Methods in Engineering **13**, 123–138 (1978).

II STRUCTURAL EQUATIONS

The elastic structure occupies a region R that is bounded by a surface ∂R. The vibration of the structure is assumed to be described by the equations of linear elasticity. Since the equations are linear, the time dependence of all variables will be expressed in terms of Fourier transforms, i.e.,

$$\mathbf{u}(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathbf{u}(x,\omega)e^{i\omega t}\, d\omega,$$

where the complex function $\mathbf{u}(x,\omega)$ is called the Fourier transform of $\mathbf{u}(x,t)$. The linearity of the equations of motion allows them to be expressed entirely in terms of the complex Fourier components. In what follows, the explicit dependence of variables on the frequency ω will usually be omitted.

In the finite element method, summarized here, the displacement $\mathbf{u}$ is approximated by a linear combination of vector interpolation functions $\boldsymbol{\phi}_m(x)$, i.e.,

$$\mathbf{u}(x) \doteq \sum_{m=1}^{M} U_m \boldsymbol{\phi}_m(x) \ , \quad x \in R. \tag{1}$$

Typically, the values U_m correspond to the components of displacement at a finite number of points in the body called nodes, and the functions $\boldsymbol{\phi}_m(x)$ form a basis for piecewise polynomial interpolation between nodes. The finite element equations can be obtained by utilizing the approximation of equation (1) in a variational formulation of the equations of elasticity[4]. The resulting finite element equations are usually expressed in the matrix form

$$(-\omega^2 M + K)U = F^L, \tag{2}$$

where the *mass matrix* M has the components

$$M_{ij} = \int_R \rho_s \boldsymbol{\phi}_i \cdot \boldsymbol{\phi}_j \, dV \tag{3}$$

$\rho_s \ldots$ mass density of material;

[4]O.C. Zienkiewicz, **The Finite Element Method**, Third Edition, McGraw-Hill, 1977, Chapter 3.

the *stiffness matrix* K has the components

$$K_{ij} = \int_R C_s[\hat{\nabla}\phi_i] \cdot \hat{\nabla}\phi_j \, dV \tag{4}$$
$$\hat{\nabla}\phi = \tfrac{1}{2}(\nabla\phi + \nabla\phi^T)$$

C_s ... elasticity tensor relating the stress tensor and strain tensor;

and the *load vector* F^L has the components

$$F^L_m = \int_{\partial R} \boldsymbol{t} \cdot \boldsymbol{\phi}_m \, dS \tag{5}$$

$\boldsymbol{t}$ ... stress vector load on ∂R.

The unknown vector U consists of all displacement components at all the nodes in R.

For a structure immersed in a fluid medium it is convenient to write the load vector F^L as

$$F^L = F^R + F^D, \tag{6}$$

where F^R is associated with the acoustic loading of the fluid medium and F^D is associated with the other known forces which drive the structure.

Equations (2) and (6) can be combined to yield

$$(-\omega^2 M + K)U = F^R + F^D. \tag{7}$$

For an ideal, nonviscous fluid medium, the stress vector $\boldsymbol{t}$ can be written in terms of the acoustic pressure, i.e.,

$$\boldsymbol{t} = -p\boldsymbol{n} \tag{8}$$

$\boldsymbol{n}$ ... unit outward normal vector (pointing into fluid).

If S is the portion of ∂R that is in contact with the fluid, then the load vector F^R can be written as follows:

$$F^R_m = -\int_S p \, \boldsymbol{\phi}_m \cdot \boldsymbol{n} \, dS. \tag{9}$$

In Section III, the governing equations for the fluid medium will be discussed. These equations will relate the pressure p in equation (9) to the normal velocity on the surface S.

III ACOUSTIC EQUATIONS

In a scattering problem one is interested in the scattered pressure field p_s due to a plane wave p_{inc} impinging on the surface S bounding the elastic structure. The total pressure p is related to the scattered pressure p_s by

$$p = p_{inc} + p_s. \tag{10}$$

In a radiation problem there is no incident wave p_{inc}, so p and p_s are the same. It is shown by Wilton[5] that the total pressure p satisfies the Helmholtz integral formulas

$$\begin{aligned}\int_S \left[p(r)\frac{\partial G(r,r')}{\partial n_r} - \frac{\partial p(r)}{\partial n_r}G(r,r')\right] dS(r) &= \frac{\alpha}{4\pi}p(r') - p_{inc}(r'), \\ &\qquad r' \text{ on } S \qquad (11a) \\ &= -p_{inc}(r'), \\ &\qquad r' \text{ interior to } S \qquad (11b) \\ &= p(r') - p_{inc}(r'), \\ &\qquad r' \text{ external to } S \qquad (11c)\end{aligned}$$

where

$$G(r,r') = \frac{e^{ik|r-r'|}}{4\pi|r-r'|}$$

is the three-dimensional free Green's function for the Helmholtz differential equation

$$\nabla^2 p + k^2 p = 0;$$

α is the solid angle at $r' \in S$ subtended by the fluid exterior to S; and $\frac{\partial}{\partial n_r}$ denotes differentiation in the outward normal direction $\boldsymbol{n}$ at r.

At any point $r \in S$ at which the normal $\boldsymbol{n}(r)$ is well-defined, the normal derivative of the pressure p is related to the normal velocity $v(r)$ by[6]

$$\frac{\partial p(r)}{\partial n_r} = -i\omega\rho v(r), \tag{12}$$

[5]D.T. Wilton, ibid, page 125.
[6]D.T. Wilton, ibid, page 126.

where ρ is the fluid density. Combination of equations (11) and (12) gives

$$\frac{\alpha}{4\pi}p(r') - \int_S p(r)\frac{\partial G(r,r')}{\partial n_r}\,dS(r) = i\omega\rho\int_S v(r)G(r,r')\,dS(r) + p_{inc}(r') \qquad (13a)$$

for r' on the surface S;

$$-\int_S p(r)\frac{\partial G(r,r')}{\partial n_r}\,dS(r) = i\omega\rho\int_S v(r)G(r,r')\,dS(r) + p_{inc}(r') \qquad (13b)$$

for r' interior to the surface S; and

$$p_s(r') = \int_S \left\{p(r)\frac{\partial G(r,r')}{\partial n_r} + i\omega\rho v(r)G(r,r')\right\} dS(r) \qquad (13c)$$

for r' exterior to the surface S.

If v and p_{inc} are prescribed on S, then equation (13a) becomes an integral equation of the second kind for p on S. This integral equation is well suited for numerical computations for most wavenumbers k, but its solution is not unique whenever k is an eigenvalue of the interior Dirichlet problem. One effective approach to overcoming this lack of uniqueness is to use the combined Helmholtz integral equation formulation[7]. In this approach, an overdetermined system of equations is formed by supplementing the integral equation (13a) with the integral equation (13b) evaluated at selected points r' interior to S.

The combination of equations (13a) and (13b) can be solved numerically by approximating p and v on S by piecewise constant functions, i.e.,

$$p(r) \doteq p_n \quad \text{and} \quad v(r) \doteq v_n \quad \text{for } r \in S_n, \qquad (14)$$

where $S_1, \ldots, S_N$ are nonoverlapping subregions of S whose union is S. Collocation of (13a) at the N surface points r_m ($m = 1, \ldots, N$) where the surface is smooth ($\alpha = 2\pi$), and (13b) at the N' interior points r_m ($m = N+1, \ldots, N+N'$) leads to the matrix relation

$$AP = BV + P_{inc}, \qquad (15)$$

[7] H. Schenck, ibid.

where

$$A_{mn} = \delta_{mn} - \int_{S_n} \frac{\partial G(r, r_m)}{\partial n_r} dS(r) \quad m = 1, \ldots, N + N' \quad n = 1, \ldots, N \tag{16}$$

$$B_{mn} = i\omega\rho \int_{S_n} G(r, r_m) \, dS(r) \quad m = 1, \ldots, N + N' \quad n = 1, \ldots, N \tag{17}$$

and

$$P = \begin{pmatrix} p_1 \\ \vdots \\ p_N \end{pmatrix}, \qquad V = \begin{pmatrix} v_1 \\ \vdots \\ v_N \end{pmatrix}, \qquad P_{inc} = \begin{pmatrix} p_{inc}(r_1) \\ \vdots \\ p_{inc}(r_{N+N'}) \end{pmatrix}.$$

Equations 15–17 represent a special case of more general relations given by Wilton[8]. In the next section, the acoustic relation (15) will be coupled with the finite element structural equations. Solution of the coupled equations will determine the surface pressures $p_1, \ldots, p_N$ and the normal velocities $v_1, \ldots, v_N$.

Applying the piecewise constant approximation to equation (13c) results in

$$p_s(r') = \sum_{n=1}^{N} p_n \int_{S_n} \frac{\partial G(r, r')}{\partial n_r} dS(r) + i\omega\rho \sum_{n=1}^{N} v_n \int_{S_n} G(r, r') \, dS(r). \tag{18}$$

Equation (18) can be used to calculate the scattered pressure p_s at any point r' exterior to S. If $r_1, \ldots, r_J$ is an arbitrary set of field points where it is desired to calculate the scattered pressure, then equation (18) reduces to the matrix equation

$$P_s = A^F P + B^F V, \tag{19}$$

where

$$A^F_{mn} = \int_{S_n} \frac{\partial G(r, r_m)}{\partial n_r} dS(r) \tag{20}$$

and

$$B^F_{mn} = i\omega\rho \int_{S_n} G(r, r_m) \, dS(r). \tag{21}$$

[8] D.T. Wilton, ibid, pages 126-127.

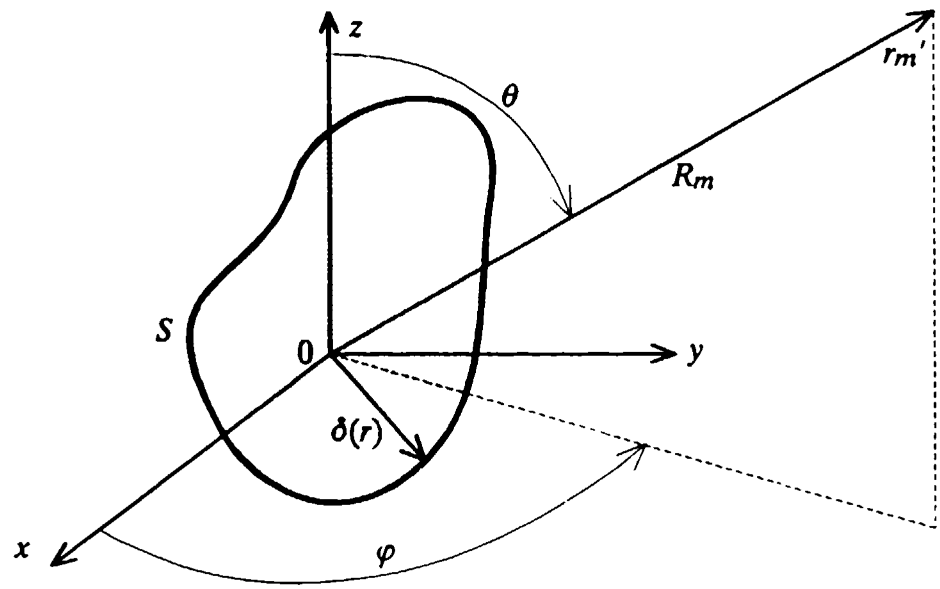

Figure 1. Far-field quantities.

Often it is desired to calculate the scattered or the radiated pressure ($P_{inc} = 0$) in the 'far-field' of S where the field pressure spreads spherically. Let us define a normalized far-field pressure p_s^{FF} by

$$P_s^{FF} = P_s / \left(\frac{e^{-ikR}}{R} \right), \tag{22}$$

where R is the distance from an origin near the surface S to the field point. It can be shown[9] that equation (19) can be approximated in the far-field by

$$P_s^{FF} = A^{FF} P + B^{FF} V, \tag{23}$$

where

$$A_{mn}^{FF} = \frac{ik}{4\pi} \int_{S_n} (\hat{\mathbf{R}}_m \cdot \boldsymbol{n}) e^{ik\hat{\mathbf{R}}_m \cdot \boldsymbol{\delta}(r)} \, dS(r) \tag{24}$$

and

$$B_{mn}^{FF} = \frac{i\omega\rho}{4\pi} \int_{S_n} e^{ik\hat{\mathbf{R}}_m \cdot \boldsymbol{\delta}(r)} \, dS(r). \tag{25}$$

The vectors $\boldsymbol{R}_m$ and $\boldsymbol{\delta}(\mathrm{r})$ are pictured in figure 1 and $\hat{\mathbf{R}}_m$ is the unit vector in the direction $\boldsymbol{R}_m$. An important point to note is that the matrices A^{FF} and B^{FF} are functions of θ and ϕ, but are independent of R.

[9] G.W. Benthien, D. Barach, and D. Gillette, ibid, Appendix A4.

The acoustic relations presented in this section have been incorporated into a computer program called CHIEF. This program was initially set up to handle only acoustic radiation problems, but it has recently been modified to handle scattering problems as well. The CHIEF program has special provisions to take advantage of one, two, or three planes of symmetry and finite degrees of rotational symmetry in the geometry and/or in the forcing function. In large problems it is very important from a cost standpoint to take advantage of any symmetry that exists.

IV COUPLED EQUATIONS

In this section it will be shown how the finite element equations of Section II and the acoustic equations of Section III can be combined in order to solve the coupled structure-acoustic problem. This approach has been implemented at NOSC and has been successfully applied to a variety of structural acoustic problems.

Making use of the piecewise constant approximation to p in equation (9), one obtains

$$F_m^R = \sum_{n=1}^{N} p_n \int_{S_n} \boldsymbol{\phi}_m \cdot \boldsymbol{n}\, dS, \quad m = 1, \ldots, M. \tag{26}$$

Equation (26) can be written in the matrix form

$$F^R = -CDP, \tag{27}$$

where

$$C_{mn} = \frac{1}{S_n} \int_{S_n} \boldsymbol{\phi}_m \cdot \boldsymbol{n}\, dS \tag{28}$$

and

$$D = \mathrm{diag}(S_1, \ldots, S_N). \tag{29}$$

It is convenient to choose the finite element break-up so that the portions of the elements lying on the wet surface S coincide with the CHIEF regions $S_1, \ldots, S_N$. If this is the case, then the matrix C can be computed within the finite element program. The n-th column of C corresponds to a load vector in which a constant pressure $-1/S_n$ is applied on S_n with zero pressure being applied to the remainder of the surface.

Since CHIEF and the finite element program use different orders of interpolation, it is not possible to fully impose the constraint for continuity of normal velocity. One approximate method for imposing this constraint is to equate the normal velocity v_n in the acoustic equations to the average of the normal component of the finite element velocity over S_n, i.e.,

$$v_n = \frac{1}{S_n} \int_{S_n} i\omega[\boldsymbol{u}(x) \cdot \boldsymbol{n}(x)]\, dS. \tag{30}$$

Substitution of the interpolation relation (1) into equation (30) gives

$$\begin{aligned} v_n &= i\omega \sum_{m=1}^{M} U_m \left(\frac{1}{S_n} \int_{S_n} \phi_m(x) \cdot \boldsymbol{n}(x)\, dS \right) \\ &= i\omega \sum_{m=1}^{M} C_{mn} U_m. \end{aligned} \tag{31}$$

Equation (31) can be written in the matrix form

$$V = i\omega C^T U. \tag{32}$$

Solution of equation (7) for U gives

$$U = (-\omega^2 M + K)^{-1} F^R + (-\omega^2 M + K)^{-1} F^D. \tag{33}$$

Substitution of (27) into (33) yields

$$U = -(-\omega^2 M + K)^{-1} CDP + (-\omega^2 M + K)^{-1} F^D. \tag{34}$$

Combination of equations (32) and (34) results in

$$V = -i\omega C^T (-\omega^2 M + K)^{-1} CDP + i\omega C^T (-\omega^2 M + K)^{-1} F^D. \tag{35}$$

Substitution of (35) into (15) gives

$$AP = -i\omega BC^T (-\omega^2 M + K)^{-1} CDP + i\omega BC^T (-\omega^2 M + K)^{-1} F^D + P_{inc}, \tag{36}$$

or equivalently,

$$[A + i\omega BC^T (-\omega^2 M + K)^{-1} CD] P = i\omega BC^T (-\omega^2 M + K)^{-1} F^D + P_{inc}. \tag{37}$$

If F^D and P_{inc} are prescribed, then P can be obtained by solving the (possibly over-determined) system of equations in (37). Once P is obtained, V can be obtained from (35).

In the CHIEF program the system of equations (37) is solved in the least squares sense using Householder transformations[10]. To solve an overdetermined system of equations

$$MX = b. \tag{38}$$

[10] P. Businger and G.H. Golub, *Linear Least Squares Solutions by Householder Transformations*, Numer. Math. 7, 269-276 (1965).

by this method, a unitary matrix Q is constructed so that

$$QM = \begin{pmatrix} U \\ 0 \end{pmatrix}, \tag{39}$$

where U is a square upper triangular matrix. The matrix Q is a product of Householder transformations. The least square solution to (38) is given by

$$X = (U^{-1}, 0)Qb. \tag{40}$$

If equation (37) is to be solved at a number of frequencies, then it is more economical to obtain the inverse $(-\omega^2 M + K)^{-1}$ in terms of normal modes. The normal modes E_m ($m = 1, \dots, L$) are solutions of the generalized eigenproblem

$$(-\omega_m^2 M + K)E_m = 0, \tag{41}$$

where $\omega_1^2, \dots, \omega_L^2$ are the associated eigenvalues. It can be shown that the normal modes are M-orthogonal, i.e.,

$$E_m^T M E_n = 0, \quad m \neq n. \tag{42}$$

The modes are usually normalized so that

$$E_m^T M E_m = 1, \quad m = 1, \dots, L. \tag{43}$$

Let E be the $L \times L$ matrix whose columns are the normal modes $E_1, \dots, E_L$. Then the relations (42) and (43) can be expressed in the single matrix relation

$$E^T M E = I. \tag{44}$$

It follows from (41) and (44) that

$$E^T K E = \Omega, \tag{45}$$

where Ω is the diagonal matrix

$$\Omega = \begin{pmatrix} \omega_1^2 & & 0 \\ & \ddots & \\ 0 & & \omega_L^2 \end{pmatrix}. \tag{46}$$

If the matrix T is defined by

$$T = (-\omega^2 M + K)^{-1}, \qquad (47)$$

then it follows that

$$(-\omega^2 M + K)T = I. \qquad (48)$$

If W is defined by

$$T = EW, \qquad (49)$$

then W satisfies

$$(-\omega^2 M + K)EW = I. \qquad (50)$$

Multiplying (50) by E^T and making use of equations (44) and (45), yields

$$(-\omega^2 I + \Omega)W = E^T. \qquad (51)$$

It follows from (51) that

$$W = (-\omega^2 I + \Omega)^{-1} E^T, \qquad (52)$$

and hence that

$$(-\omega^2 M + K)^{-1} = T = E(-\omega^2 I + \Omega)^{-1} E^T. \qquad (53)$$

Since $-\omega^2 I + \Omega$ is diagonal, its inverse at each frequency is simple to compute. Substitution of (53) into (37) gives

$$[A + i\omega BC^T E(-\omega^2 I + \Omega)^{-1} E^T CD]P = i\omega BC^T E(-\omega^2 I + \Omega)^{-1} E^T F^D + P_{inc}. \quad (54)$$

If X is defined by

$$X = E^T C, \qquad (55)$$

then equation (56) can be written as

$$[A + i\omega BX^T(-\omega^2 I + \Omega)^{-1} XD]P = i\omega BX^T(-\omega^2 I + \Omega)^{-1} E^T F^D + P_{inc}. \qquad (56)$$

Given F^D and P_{inc}, equation (54) can be solved for P.

The solution of the coupled problem can be summarized as follows:

1. Compute the matrices M, K, and C using the finite element program.

2. Calculate the matrix E of normal modes and the diagonal matrix Ω of characteristic frequencies by solving the generalized eigenproblem

$$(-\omega_m^2 M + K)E_m = 0, \quad m = 1, \ldots, L.$$

3. Generate the matrix $X = E^T C$.

4. Compute the surface matrices A, B and the far-field matrices A^{FF}, B^{FF} using CHIEF.

5. Form and solve the system of equations (56) for P at each frequency.

6. Compute the normal velocity V at each frequency using equation (35).

7. Solve for the scattered far-field pressure vector P_s^{FF} at each frequency using equation (23), i.e.,

$$P_s^{FF} = A^{FF} P + B^{FF} V.$$

V SCATTERING PROBLEMS

In a pure acoustical scattering problem, the mechanical or electromechanical load vector F^D is zero so that the forced vibrations of the structure are caused entirely by an incident pressure wave p_{inc}. In most cases p_{inc} is taken to be a plane wave arriving from a particular direction, but the formulation used in this report can be used to calculate the scattering due to any incident field

The fundamental quantity of interest is the scattered pressure p_s, which is defined as the difference between the total pressure p and the incident pressure p_{inc}. It is usually desired to know the scattered pressure on the surface and in the far-field of the object as well as the velocity or displacement on the surface of the object.

The basic computation procedure follows the eight steps outlined in Section IV with F^D set equal to zero. This procedure produces the surface pressures, forces, velocities, and displacements as well as the scattered pressures exterior to the object. This method accounts for the complete interaction of the incident acoustic field with the elastic structure to the extent that the structure is accurately represented by the finite element model and that the acoustic field is accurately represented by CHIEF.

The method can be easily adapted to the simpler problem of scattering from a rigid object ($V = 0$). In this case, it follows from equation (15) that $P = A^{-1}P_{inc}$. Thus, the scattered pressure field can be computed as in Section IV with $P = A^{-1}P_{inc}$ and $V = 0$.

Typically, one normalizes the scattered pressure by the incident pressure and calculates either the *target strength*, the *form function*, or the *scattering cross section*. The *target strength* is defined by

$$\text{Target Strength } (TS) \equiv 10\log_{10}\left(\frac{\text{Scattered Intensity}}{\text{Incident Intensity}}\right), \tag{57}$$

where the *scattered intensity* I_s is the far-field intensity of the spherically spreading scattered wave normalized back to one meter (or one yard), i.e.,

$$
\begin{aligned}
I_s &= \lim_{R\to\infty} R^2 I(R) \\
&= \lim_{R\to\infty} \frac{R^2|P_s(R)|^2}{\rho c} \\
&= \frac{|P_s^{FF}(\theta,\phi)|^2}{\rho c},
\end{aligned} \tag{58}
$$

and, for plane-wave incidence, the incident intensity I_{inc} is given by

$$
I_{inc} = \frac{|P_{inc}(\theta_{inc},\phi_{inc})|^2}{\rho c}. \tag{59}
$$

Combining (58) and (59) with (57), gives

$$
TS = 20\log_{10}\left[\frac{|P_s^{FF}(\theta,\phi)|}{|P_{inc}(\theta_{inc},\phi_{inc})|}\right]. \tag{60}
$$

The quantity TS in equations (57) and (60) is often referred to as the *bistatic target strength* whereas the particular case where $(\theta,\phi) = (\theta_{inc},\phi_{inc})$ is called the *monostatic target strength.*

Sometimes a linear rather than a logarithmic measure of scattering strength is desired. One such measure is the *scattering cross section* σ which is related to *target strength* by

$$
TS = 10\log_{10}\left[\frac{\sigma(\theta,\phi,\theta_{inc},\phi_{inc})}{4\pi}\right], \tag{61}
$$

or equivalently

$$
\sigma(\theta,\phi,\theta_{inc},\phi_{inc}) = 4\pi\left|\frac{P_s^{FF}(\theta,\phi)}{P_{inc}(\theta_{inc},\phi_{inc})}\right|^2. \tag{62}
$$

Another commonly used linear scattering measure is the *form function* f defined by

$$
\begin{aligned}
|f(\theta,\phi,\theta_{inc},\phi_{inc})| &= \sqrt{\frac{\sigma}{\sigma_g}} \\
&= \sqrt{\frac{4\pi}{\sigma_g}}\left|\frac{P_s^{FF}(\theta,\phi)}{P_{inc}(\theta_{inc},\phi_{inc})}\right|,
\end{aligned} \tag{63}
$$

where σ_g is the geometric (high-frequency) scattering cross section of the rigid object. Thus, for a sphere

$$|f(\theta,\phi,\theta_{inc},\phi_{inc})| = \frac{2}{a}\left|\frac{P_s^{FF}(\theta,\phi)}{P_{inc}(\theta_{inc},\phi_{inc})}\right|. \tag{64}$$

All of the above expressions are valid for the general three-dimensional bistatic case where $(\theta_{inc},\phi_{inc})$ is an arbitrary incidence direction and (θ,ϕ) is an arbitrary direction of observation. Frequently it is of interest to compute the so-called monostatic (back-scattered) target strength where $\theta = \theta_{inc}$ and $\phi = \phi_{inc}$.

As an example problem, consider the scattering from a hollow steel shell immersed in water and having the following parameter values:

- Mean Radius, $a = 5\ m$
- Thickness, $h = 0.15\ m$
- Young's Modulus, $E = 2.07 \times 10^{11}\ N/m^2$
- Poisson's Ratio, $\nu = 0.3$
- Shell Density, $\rho_s = 7669\ Kg/m^3$
- Fluid Density, $\rho_w = 1000\ Kg/m^3$
- Fluid Sound Speed, $c_w = 1524\ m/sec$
- Structural Loss Factor, $\eta = 0,\ 0.01,\ 0.05,\ 0.1$.

An analytical solution to this problem is given in Junger and Feit[11]. This solution can be expressed in the present notation as follows

$$\begin{aligned}|f(\theta,\phi)| &= \frac{2}{ka}\left|\sum_{n=0}^{\infty}\frac{(2n+1)P_n(\cos\theta)}{h_n^{(2)\prime}(ka)}\right.\\ &\qquad\left.\times\left[j_n'(ka) - \frac{\rho_w c_w}{(ka)^2 h_n^{(2)\prime}(ka)(Z_n+z_n)}\right]\right|,\end{aligned} \tag{65}$$

[11] Junger and Feit, **Sound, Structures, and Their Interaction**, Second Edition, MIT Press, 1986, p. 354.

where the structural modal impedance Z_n is given by

$$Z_n = i\rho_s c_p \frac{h}{a} \frac{[\Omega^2 - (\Omega_n^{(1)})^2][\Omega^2 - (\Omega_n^{(2)})^2]}{\Omega^3 - (n^3 + n - 1 + \nu)(1 + \beta^2)}, \tag{66}$$

and the acoustical modal impedance z_n is given by

$$z_n = i\rho_w c_w \frac{h_n^{(2)}(ka)}{h_n^{(2)\prime}(ka)}. \tag{67}$$

Here $(\Omega_n^{(1)})^2$ and $(\Omega_n^{(2)})^2$ are the roots of the quadratic equation in Ω^2

$$\begin{aligned}
&\Omega^4 - [1 + 3\nu + \lambda_n - \beta^2(1 - \nu - \lambda_n^2 - \nu\lambda_n]\Omega^2 \\
&+(\lambda_n - 2)(1 - \nu^2) + \beta^2[\nu_n^3 - 4\lambda_n^2 + \lambda_n(5 - \nu^2) - 2(1 - \nu^2)] \\
&= 0,
\end{aligned}$$

and

$$\begin{aligned}
\lambda_n &= n(n+1) \\
\beta^2 &= h^2/(12a^2) \\
c_p &= \sqrt{E/[\rho_s(1 - \nu^2)]} \\
\Omega &= \omega a / c_p.
\end{aligned} \tag{68}$$

A description of the numerical model set-up for this problem is given below.

1. A spherical coordinate system was used having its center at the center of the sphere and its polar axis in the direction of the incident wave. Thus, all fields are axisymmetric in the ϕ-direction.

2. Forty quadratic axisymmetric thick shell elements were used in the finite element (MARTSAM) calculations.

3. The CHIEF computations were made with 10 rotational symmetry blocks having one element per block in the ϕ-direction and 40 elements per block in the θ-direction. The Gaussian quadrature order was 16 in the ϕ-direction and four in the θ-direction.

4. Since the form function varies rapidly with frequency when the structural damping is low, a very fine resolution in frequency was used. The increment in ka was 0.01 as ka varied from 1.5 to 3.09.

Figure 2 shows a comparison of the numerical and the series solution for the form function in the back-scatter direction versus ka when there is no structural damping ($\eta = 0$). The accuracy is quite good. Figure 3 shows a similar comparison in the forward scatter direction. Figures 4 through 6 show how the frequency response of the back-scatter form function smooths out as the damping coefficient η takes on the values 0.01, 0.05, 0.1. Damping is included in the models by making the Young's Modulus of the shell complex, i.e., $E = E_r(1 + i\eta)$. Since the shell in this problem is composed of a single material, the finite element stiffness matrix is proportional to E. Therefore, the eigenvectors are independent of η and the eigenvalues are given by $-\omega_n^2/(1 + i\eta)$ where ω_n^2 is an eigenvalue for the case $\eta = 0$. Figure 7 is like figure 6 except for the forward scatter direction. Finally, figure 8 shows how the target strength varies with angle for $ka = 2.37$ and $\eta = 0$.

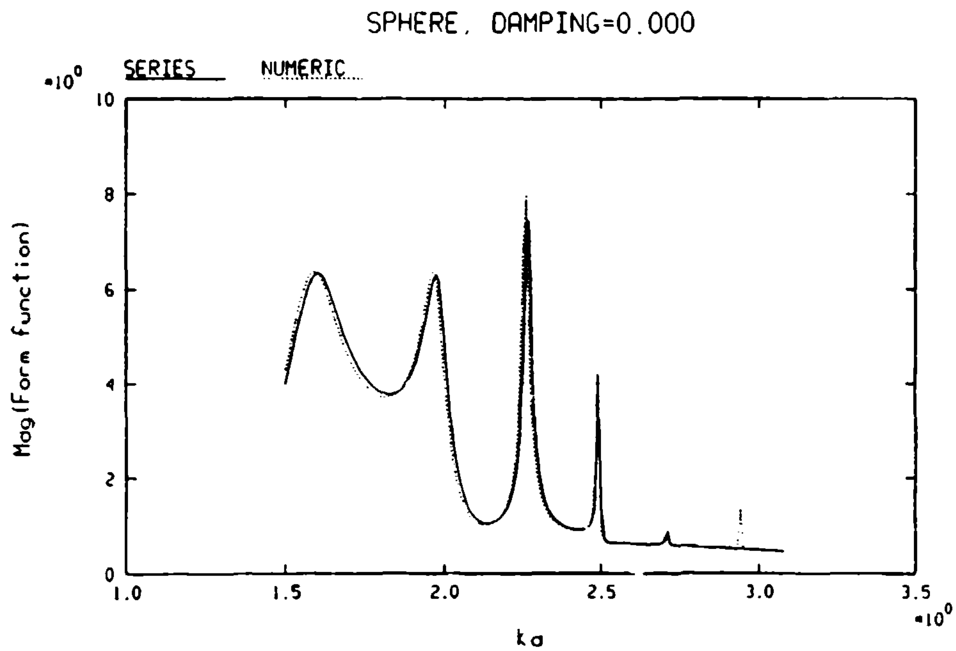

Figure 2. Form function in the back-scatter direction ($\eta = 0$).

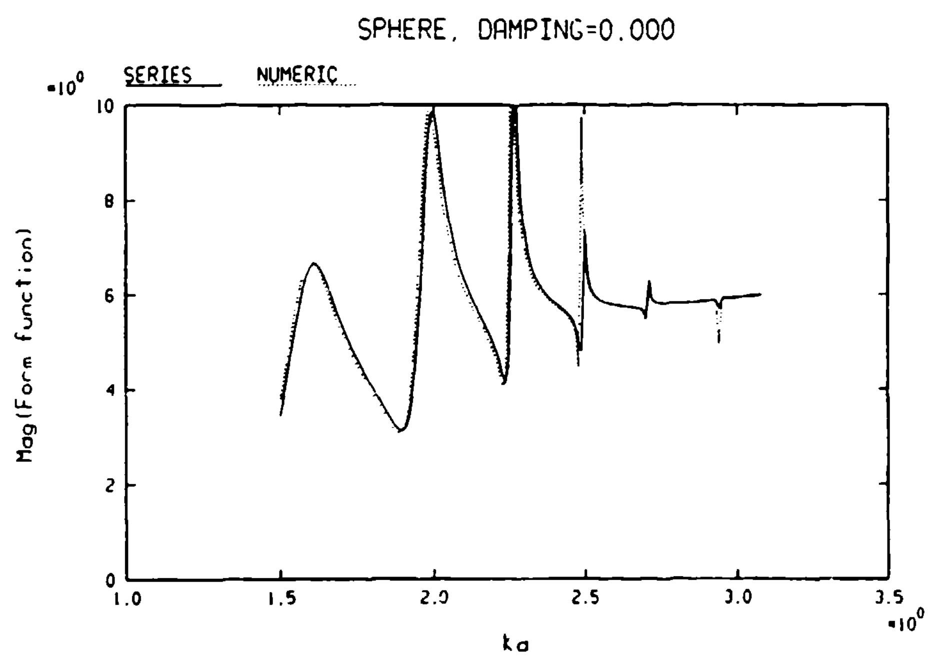

Figure 3. Form function in the forward-scatter direction ($\eta = 0$).

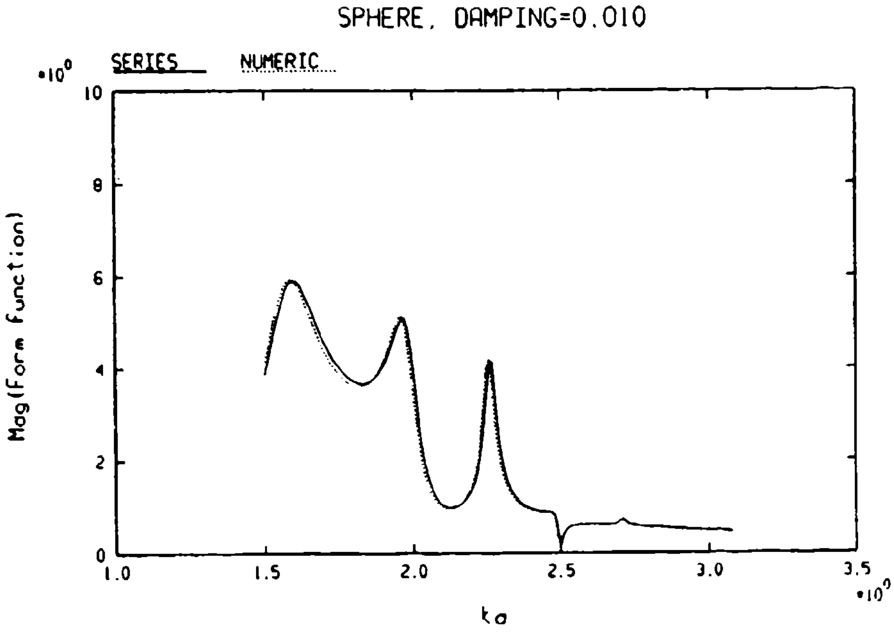

Figure 4. Form function in the back-scatter direction ($\eta = 0.01$).

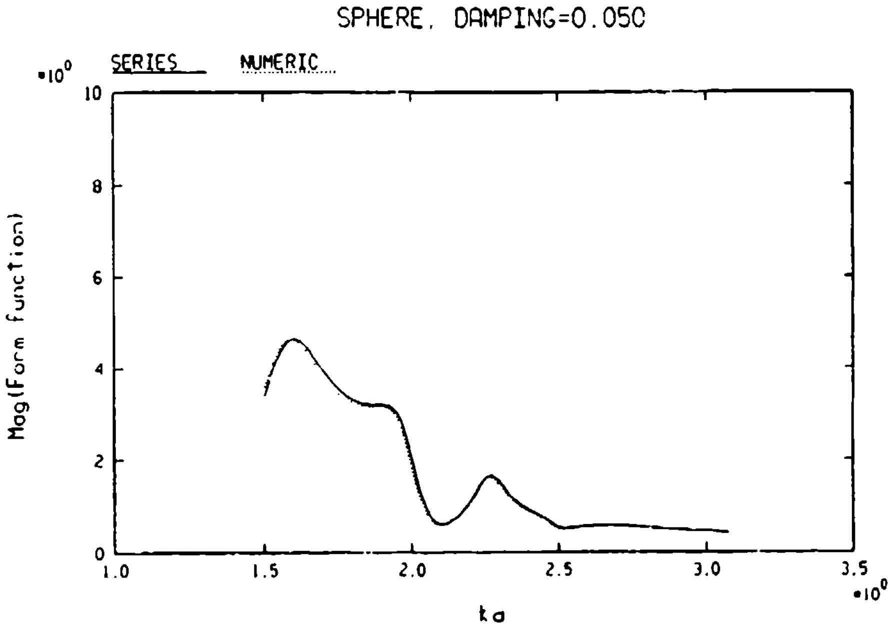

Figure 5. Form function in the back-scatter direction ($\eta = 0.05$).

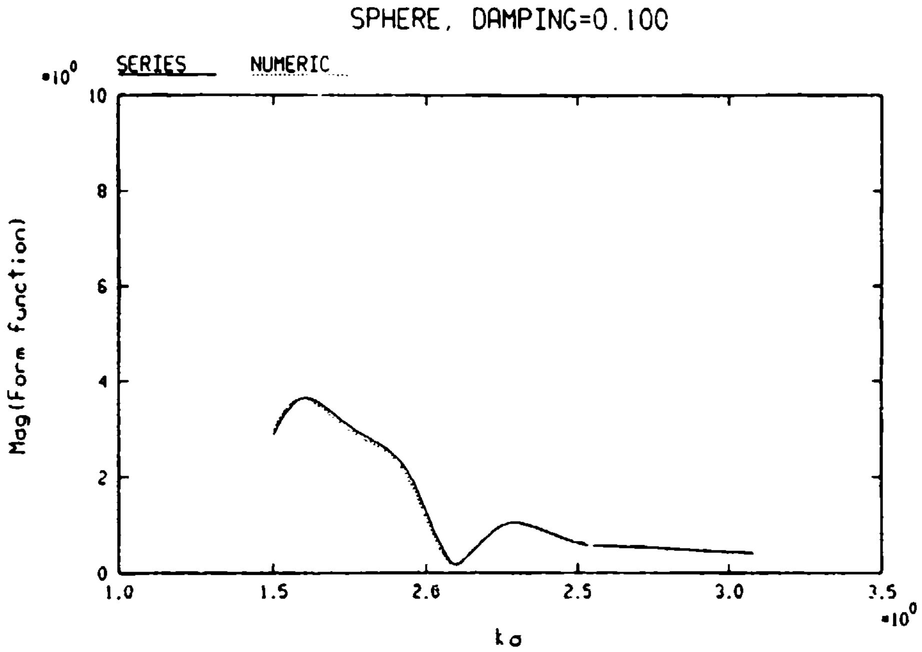

Figure 6. Form function in the back-scatter direction ($\eta = 0.1$).

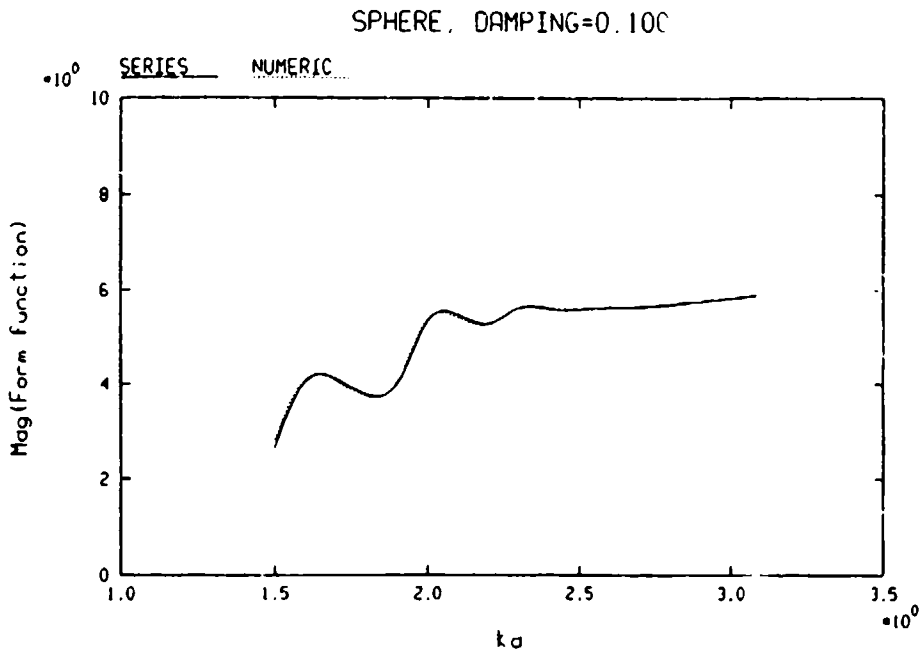

Figure 7. Form function in the forward-scatter direction ($\eta = 0.1$).

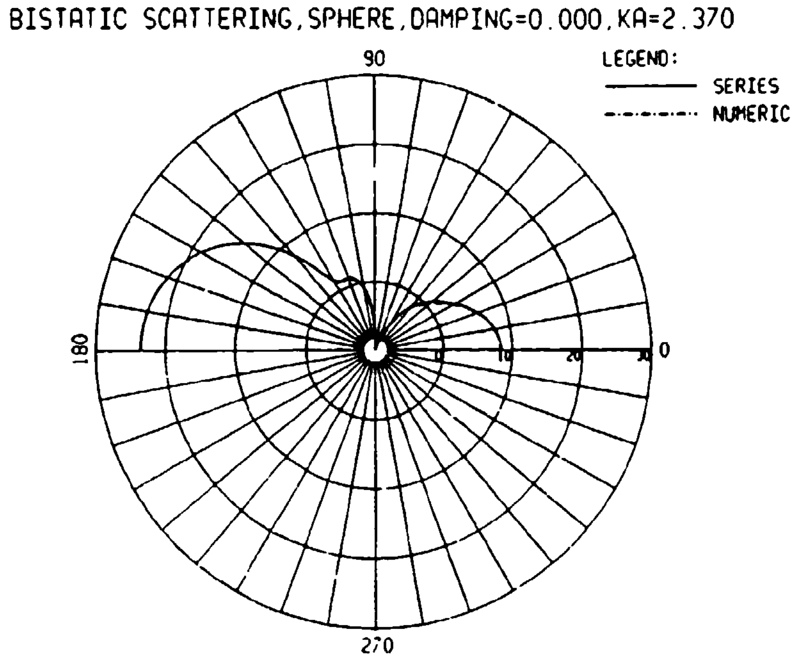

Figure 8. Target strength versus angle ($ka = 2.37$, $\eta = 0$).

VI RADIATION PROBLEMS

In a structural radiation problem the incident pressure p_{inc} is taken to be zero and the structure is forced to vibrate by a prescribed mechanical or electromechanical load vector F^D. Again the basic computational steps outlined in Section IV are followed with $P_{inc} = 0$. Since $P_{inc} = 0$, the formulas for scattered pressure now apply to the total pressure p as well.

To assess the directional response of a radiating structure one usually plots the quantity

$$K(\theta,\phi) = 10\log_{10}\left|\frac{p^{FF}(\theta,\phi)}{p^{FF}(\theta_0,\phi_0)}\right|^2,$$

where (θ_0, ϕ_0) is some prescribed reference direction which is usually taken to be the direction of maximum response. The quantity $K(\theta,\phi)$ is called the *directivity pattern* of the radiator.

The *source level* (SL) of the radiator is defined by

$$\begin{aligned} SL &= 10\log_{10}\left[\lim_{R\to\infty}\left|\frac{Rp(R,\theta_0,\phi_0)}{1\ \mu\ \mathrm{Pa}}\right|^2\right] \\ &= 10\log_{10}\left|\frac{p^{FF}(\theta_0,\phi_0)}{10^{-6}\ \mathrm{Pa}}\right|^2. \end{aligned} \tag{69}$$

The *total radiated power* W is given by

$$W = \int_{S_R} I\,dS, \tag{70}$$

where I is the intensity and S_R is a spherical surface of radius R which completely surrounds the radiator. The *directivity factor* D is defined by

$$D = \frac{I(R,\theta_0,\phi_0)}{\dfrac{1}{4\pi R^2}\displaystyle\int_{S_R} I\,dS}. \tag{71}$$

In the far-field we have

$$I(R, \theta_0, \phi_0) = \frac{|p(R, \theta_0, \phi_0)|^2}{\rho_w c_w} = \left| \frac{p^{FF}(\theta_0, \phi_0)}{\rho_w c_w R} \right|^2 . \tag{72}$$

Combination of equations (69)-(71) gives

$$SL = 10 \log_{10} \left(\frac{\rho_w c_w}{4\pi \cdot 10^{-12}\ \mathrm{Pa}^2} \right) + 10 \log_{10} W + DI, \tag{73}$$

where $DI = 10 \log_{10} D$. The quantity DI is called the *directivity index*. If $\rho_w c_w$ is taken to be 1.5×10^6 which is a typical value for water, then

$$10 \log_{10} \left(\frac{\rho_w c_w}{4\pi \cdot 10^{-12}\ \mathrm{Pa}^2} \right) = 170.8,$$

and hence equation (73) becomes

$$SL = 170.8 + 10 \log_{10} W + DI. \tag{74}$$

The source level can be computed using equation (69) and the total radiated power can be computed from

$$W = \int_S \mathrm{Re}(pv^*)\, dS = \sum_{m=1}^{M} S_m \mathrm{Re}(p_m v_m^*), \tag{75}$$

where p_m and v_m are the rms pressures and normal velocities on the m-th CHIEF subdivision. Therefore, the directivity index DI can be computed using equation (74), i.e.,

$$DI = SL - 10 \log_{10} W - 170.8\ . \tag{76}$$

As an example problem consider the same spherical shell that was used in the example scattering problem of Section V. This time the incident field P_{inc} will be set to zero and the shell will be driven by a constant unit pressure applied over a polar cap having a half-angular width of 4.5 degrees. An analytic series solution for the far-field pressure is given by

$$p(R, \theta, \phi) = \frac{\rho c}{2} \frac{e^{-ikR}}{kR} \sum_{n=0}^{\infty} \frac{i^n [P_{n-1}(\cos\theta_0) - P_{n+1}(\cos\theta_0)]}{h_n^{(2)\prime}(ka)(Z_n + z_n)} P_n(\cos\theta), \tag{77}$$

where $h_n^{(2)}$, Z_n and z_n are the same as in the scattering problem and θ_0 is the angular extent of the driven spherical cap.

Figure 9 shows a plot of source level versus ka ($\eta = 0$) for both the analytic series solution and the numerical solution.

Again the agreement is quite good. Figure 10 shows the same comparison for the case of high structural damping ($\eta = 0.1$). Finally, figures 11 and 12 show polar plots of the normalized far-field pressure versus angle for a ka of 2.37 and a ka of 3.0. Notice that the shape of the patterns changes dramatically with frequency due to the resonant structure of the shell.

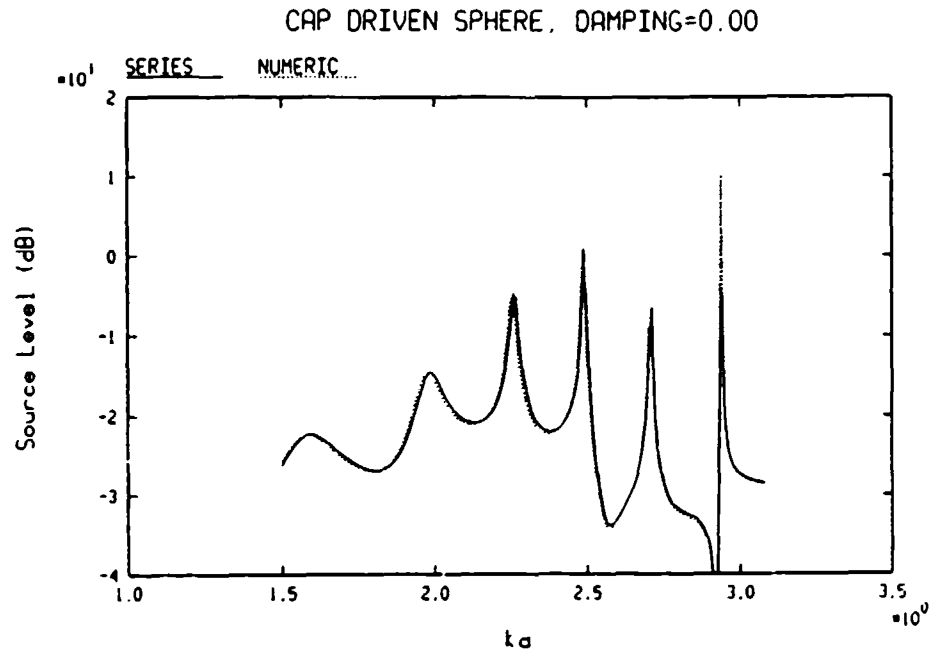

Figure 9. Source level versus ka ($\eta = 0$).

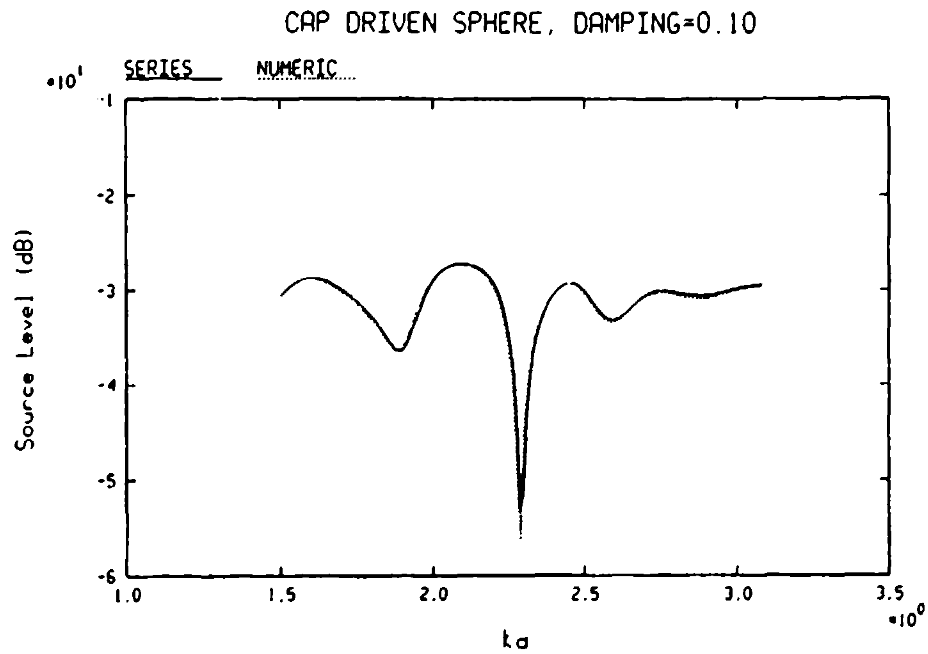

Figure 10. Source level versus ka ($\eta = 0.1$).

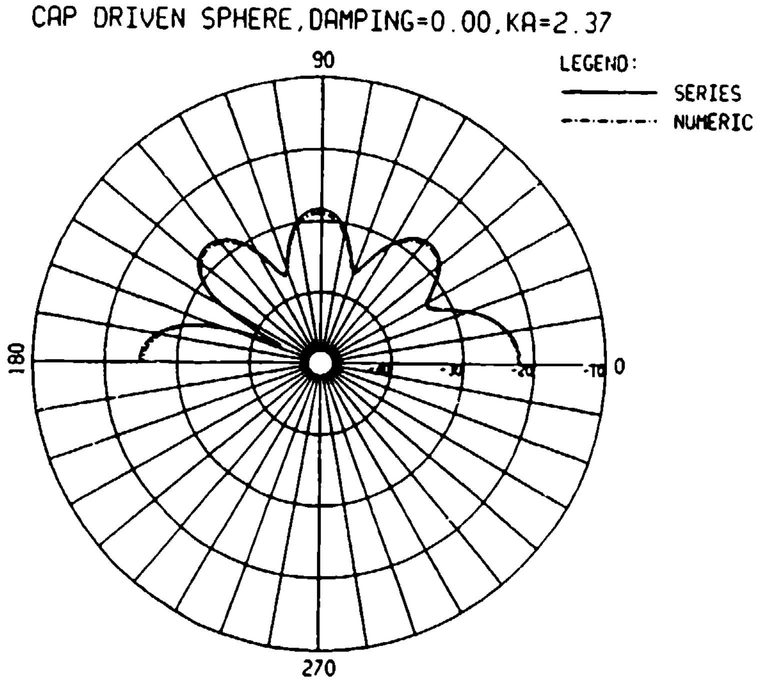

Figure 11. Normalized far-field pressure versus angle ($ka = 2.37$).

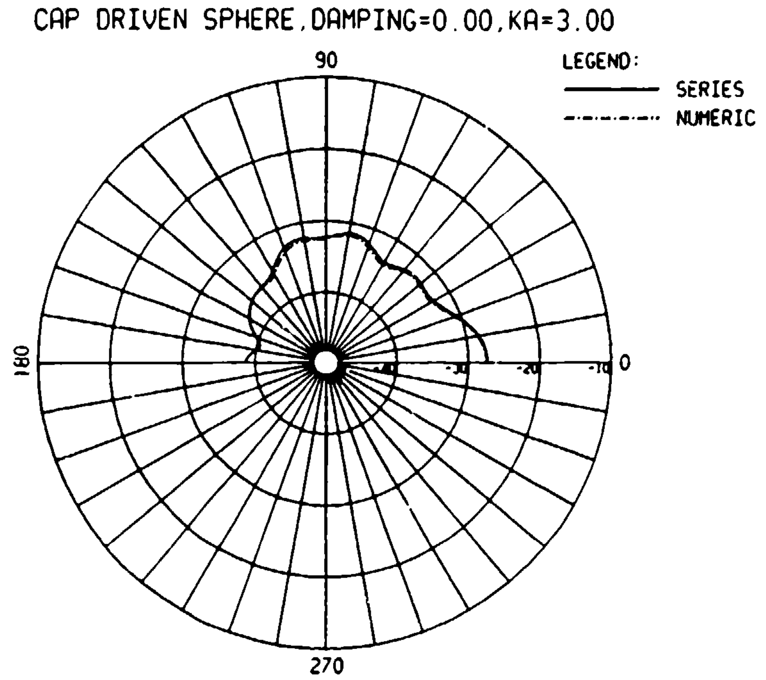

Figure 12. Normalized far-field pressure versus angle ($ka = 3.0$).

VII SUMMARY

A general method for solving acoustic radiation and scattering problems in which there is significant interaction and effect of the fluid medium on the motion of an elastic object has been developed and described mathematically. The method can be described in three major parts:

1. A finite element model (MARTSAM), which was developed to solve complicated structural problems based on the fundamental equations of linear elasticity;

2. An acoustic radiation and scattering method (CHIEF), which is based on an integral representation of the solution of the Helmholtz wave equation for a homogeneous fluid;

3. Additional programmed logic to couple these programs together in an efficient computational scheme to calculate the principal surface and field quantities of interest in a radiation or scattering problem.

The method is capable of dealing with elastic objects which can be of relatively arbitrary shape and can have complex internal elastic structure. Structural damping can be included by assigning a (homogeneous) damping factor to the elastic material. The forcing function can be an arbitrary incident wave or a mechanical or electromechanical force or a combination of forces.

A major emphasis of this report is to cast the governing equations and relationships in a computationally efficient form to allow the spatial and spectral response to be calculated with high resolution. (Nevertheless, the fact that the equations are discretized in terms of numerous areas which are small in comparison with the wavelength implies that the computational time and cost is a steep function of frequency, and the method is fundamentally a low-frequency method.) Other factors which have been carefully considered in developing the computational algorithms are the potential symmetries inherent in the object and/or the forcing terms, and the ability to save major intermediate results for recalculation of similar problems with different parameters.

Surface pressures, displacements, and velocities as well as field pressures can be determined. For scattering problems, the output can also consist of bistatic or monostatic target strengths, scattering cross sections, or form functions. For radiation problems, the source level, directivity patterns, directivity index, and the total radiated power can all be calculated, as well as intermediate results such as radiation impedance.

The accuracy of the method has been illustrated by calculating both radiation and scattering from a submerged spherical shell over the frequency range where the size of the object is comparable with the acoustic wavelength and where significant interaction and resonant effects are exhibited. The numerical results from this method compare favorably with an analytic (series) solution for this canonical problem.

The method was developed, of course, to obtain accurate and high resolution results in much more general or complicated problems which are not analytically tractable. In fact, it has been applied with success to complex transducer array problems and to determining the target strength of realistic submarine-like structures at low frequencies. Results of these practical Navy problems of interest are not contained in this report but will be reported separately in appropriate reports and publications.